# Fall 2015 Calculus Final Exam Review

Section I	Part A: 16 Problems	No Calc.	36 min
MC	Part B: 8 Problems	Calc Allowed	25 min
Section II	Part A: 1 Problem	Calc. Allowed	15 min
FR	Part B: 2 Problems	No Calc	30 min

The use of a calculator is not permitted in this part of the exam.

$$1. \int_0^1 x^{2/3} dx$$

- - (A) 0

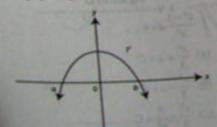
(D) - ∞

- (E) 00
- (C) 5
- 3. What is the  $\lim_{x\to -2} f(x)$ , if

$$f(x) = \begin{cases} |x-1| & \text{if } x > -2, \\ 2x+7 & \text{if } x \le -2. \end{cases}$$

- (A) -3 (B) 1
- (C) 3

- (D) 11 (E) Nonexistent
- 4. The graph of f' is shown in Figure 2T-1.



$$5. \int_{\pi/2}^{s} 2\cos t \, dt =$$

- (A) 2 cos x
- (B) -2 cos x (C) 2 sin x
- (D)  $-2\sin x + 2$  (E)  $2\sin x 2$
- 6. Given the equation  $y = 3e^{-2x}$ , what is an equation of the normal line to the graph at  $x = \ln 2$

(A) 
$$y = \frac{2}{3}(x - \ln 2) + \frac{3}{4}$$

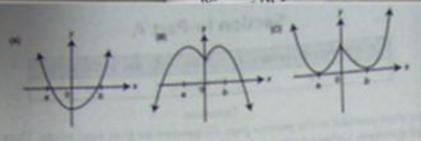
(B) 
$$y = \frac{2}{3}(x + \ln 2) - \frac{3}{4}$$

(C) 
$$y = -\frac{3}{2}(x - \ln 2) + \frac{3}{4}$$

(D) 
$$y = -\frac{3}{2}(x - \ln 2) - \frac{3}{4}$$

(E) 
$$y=24(x-\ln 2)+12$$

- (E) Undefined



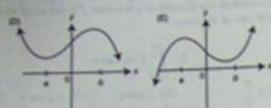


Figure 2T-2

8. If f(x) is an antiderivative of  $x^2\sqrt{x^2+1}$  and f(2)=0, then f(0)=

(D) 
$$\frac{-52}{9}$$

 If a function f is continuous for all values of x, which of the following statements in/are always true?

$$1. 2 \int_{a}^{b} f(s) ds = \int_{2a}^{2a} f(s) ds$$

II. 
$$\int_a^b f(x)dx = \int_a^a - f(x)dx$$

III. 
$$\left| \int_{a}^{b} f(x) dx \right| = \int_{a}^{b} |f(x)| dx$$

- (A) I only
- (B) I and II only
- (C) II only
- (D) II and III only

- (D) f(0) and f'(0)
- (E) f'(0) and f'(0)

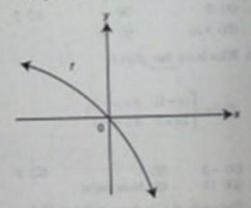


Figure 2T-3

11. 
$$\int \frac{x^4 - 1}{x^2} dx = \frac{1}{x^3}$$

$$(X) \frac{x^3}{3} + x + C$$

- 12. If p'(x) = q(x) and q is a continuous function for all values of x, then  $\int_{-1}^{0} q(4x) dx$  is
  - (A) p(0) p(-4)
- (B) 4p(0)-4p(-4)
  - (C)  $\frac{1}{4}p(0) \frac{1}{4}p(-4)$
  - (D)  $\frac{1}{4}p(0) + \frac{1}{4}p(-4)$
  - (E) p(0) + P(-4)
- 13. Water is leaking from a tank at a rate represented by f(t) whose graph is shown in Figure 2T-4. Which of the following is the best approximation of the total amount of water leaked from the tank for 1 ≤ t ≤ 3?

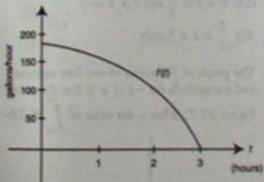


Figure 2T-4

- (A) 9 gallons
- (B) 5 gallons
- (C) 175 gallons
- (D) 350 gallons
- (E) 450 gallons
- 14. If  $f(x) = 5\cos^2(\pi x)$ , then  $f'(\frac{\pi}{2})$  is
  - (A) 0
- (B) -5
- (C) 5

- (D) -10
- (E) 10
- 15.  $g(x) = \int_{1}^{x} \frac{3t}{t^{3} + 1} dt$ , then g'(2) is
  - (A) 0
- (B)  $-\frac{2}{3}$
- (C) 3

- (D)  $\frac{-5}{6}$
- (E) =
- 16. If  $\int_{k}^{3} (2x-2) dx = -3$ , a possible value of k is
  - (A) -2
- (B) 0
- (C) 1

- (D) 2
- (E) 3
- 17. If  $\int_0^x f(x)dx = -\int_{-a}^0 f(x)dx$  for all positive values of a, then which of the following could be the graph of f? (See Figure 2T-5.)
  - (0)

13. Water is leaking from a tank at a rate represented by f(t) whose graph is shown in Figure 2T-4. Which of the following is the best approximation of the total amount of water leaked from the tank for  $1 \le t \le 3$ ?

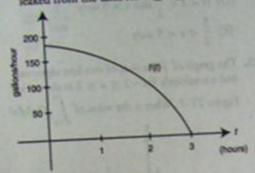
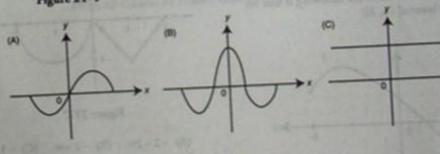


Figure 2T-4

- di, then g'(2) is

  - (2x-2)dx = -3, a possible value of k is (B) 0 (C) 1
    - (D) 2
- (E) 3
- 17. If  $\int_0^x f(x)dx = -\int_{-x}^x f(x)dx$  for all positive values of a, then which of the following could be the graph of f? (See Figure 2T-5.)



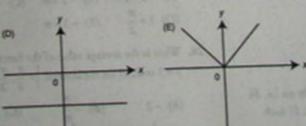


Figure 2T-5

18. A function f is continuous on [1, 5] and some of the values of f are shown below:

x	1	3	5
f(x)	-2	ь	-1

If f has only one root, r, on the closed interval [1, 5], and  $r \neq 3$ , then a possible value of b is

- (C) 1

- 19. Given the equation  $V = \frac{1}{r}\pi r^2(5-r)$ , what is the instantaneous rate of change of V with respect to r at r = 5?
  - (A)  $-\frac{25\pi}{3}$  (B)  $\frac{25\pi}{3}$  (C)  $\frac{50\pi}{3}$

- 20. What is the slope of the tangent to the curve  $x^3 - y^2 = 1$  at x = 1?

- (E) Undefined
- 21. The graph of function f is shown in Figure 2T-6. Which of the following is true for

- (A) I only
  - (B) II only
  - (C) I and II only
  - (D) II and III only
  - (E) I, II and III
- 22. The velocity function of a moving particle on the x-axis is given as  $v(t) = t^2 - 3t - 10$ . For what positive values of t is the particle's speed increasing?

(A) 
$$0 < t < \frac{3}{2}$$
 only

- (B)  $t > \frac{3}{2}$  only
- (D)  $0 < t < \frac{3}{2}$  and t > 5 only
- (E)  $\frac{3}{2} < t < 5$  only
- 23. The graph of f consists of two line segments and a semicircle for  $-2 \le x \le 2$  as shown in Figure 2T-7. What is the value of  $\int f(x)dx$ ?



- 20. What is the slope of the tangent to the curve  $x^3 y^2 = 1$  at x = 1?
- (B) 0

- (E) Undefined
- 21. The graph of function f is shown in Figure 2T-6. Which of the following is true for f on the interval (a, b)?

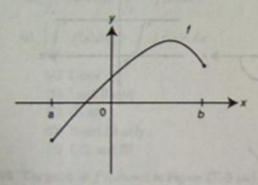


Figure 2T-6

- I. The function f is differentiable on (a, b).
- II. There exists a number k on (a, b) such that f'(k)=0.
- III.  $f^* > 0$  on (a, b).

23. The graph of f consists of two line segments and a semicircle for  $-2 \le x \le 2$  as shown in Figure 2T-7. What is the value of  $\int f(x)dx$ ?

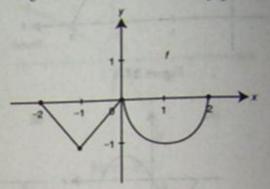


Figure 2T-7

- (A)  $-2-2\pi$  (B)  $-2-\pi$  (C)  $-1-\frac{\pi}{2}$
- (D)  $1 + \frac{\pi}{2}$
- (E)  $-1-\pi$
- 24. What is the average value of the function  $y=3\cos(2x)$  on the interval |-

  - (A) -2  $(B) -\frac{2}{}$
- (D)  $\frac{1}{\pi}$  (E)

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- 25. If  $f(x) = |x^3|$ , what is the value of  $\lim_{x \to -1} f'(x)$ ?
  - (A) -3 (B) 0

- (D) 3 (E) Undefined
- 26. A spherical balloon is being inflated. At the instant when the rate of increase of the volume of the sphere is four times the rate of increase of the radius, the radius of the sphere is

- 27. If  $\frac{dy}{dx} = \frac{x^2}{y}$  and at x = 0, y = 4, a solution to the differential equation is
- 28. The area of the region enclosed by the graph of  $x = y^2 - 1$  and the y-axis is

- (E)  $\frac{8}{3}$

# Section I-Part B

Number of Questions	Time	Use of Calculator
Number of Guestions	50 Minutes	Yes
17	JO Millione	The second secon

### Directions:

Use the same answer sheet from Part A. Please note that the questions begin with number 76. This is not an error. It is done to be consistent with the numbering system of the actual AP Calculus AB Exam. All questions are given equal weight. There is no penalty for unanswered questions. Unless otherwise indicated, the domain of a function f is the set of all real numbers. If the exact numerical value does not appear among the given choices, select the best approximate value. The use of a calculator is permitted in this part of the exam.

76. The graph of f', the derivative of f, is shown in Figure 2T-8. At which value of x does the graph f have a horizontal tangent?

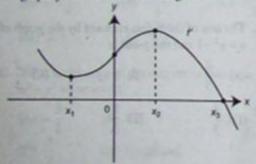


Figure 2T-8

- (A) x;
- (B) 0
- (C) x2

- (D) x1 and x2
- (E) x,
- 77. The position function of a moving particle is  $s(t) = 5 + 4s t^2$  for  $0 \le t \le 10$  where s is in

x	4	6	8	10
f(x)	2	2.4	2.8	3.2

Using three right endpoint rectangles of equal length, what is the approximate value of

$$\int_4^{10} f(x) dx?$$

- (A) 8.4
- (B) 9.6

- (D) 16.8
- (E) 20.8
- 80. Given a differentiable function f with f(-1) = 2 and  $f'(-1) = \frac{1}{2}$ . Using a tangent

line to the graph of f at x=-1, find an approximate value of f(-1.1)?

- (A) -3.05
- (B) -1.95
- (C) 0.95

(C) 14.4

- (D) 1.95
- (E) 3.05

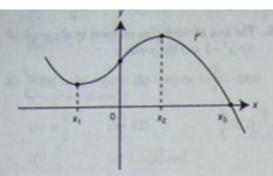


Figure 2T-8

- (A) x1
- (B) 0
- (C) x2

- (D) x1 and x2
- (E) x<sub>3</sub>
- 77. The position function of a moving particle is  $s(t) = 5 + 4t t^2$  for  $0 \le t \le 10$  where s is in meters and t is measured in seconds. What is the maximum speed in m/sec of the particle on the interval  $0 \le t \le 10$ ?
  - (A) -16
- (B) 0
- (C) 2

- (D) 4
- (E) 16
- 78. How many points of inflection does the graph of  $y = \cos(x^2)$  have on the interval  $(0, \pi)$ ?
  - (A) 0 (D) 3
- (B) 1 (E) 4
- (C) 2
- 79. Let f be a continuous function on [4, 10] and have selected values as shown below:

Using three right endpoint rectangles of equal length, what is the approximate value of

$$\int_4^{10} f(x) dx?$$

- (A) 8.4
- (B) 9.6
- (C) 14.4

- (D) 16.8
- (E) 20.8
- 80. Given a differentiable function f with f(-1)=2 and  $f'(-1)=\frac{1}{2}$ . Using a tangent line to the graph of f at x=-1, find an approximate value of f(-1.1)?
  - (A) -3.05
- (B) -1.95
- (C) 0.95

- (D) 1.95
- (E) 3.05
- 81. If area under the curve of  $y = \frac{\ln x}{x}$  is 0.66 from x = 1 to x = b, where b > 1, then the value of b is approximately,
  - (A) 1.93
- (B) 2.25
- (C) 3.15

- (D) 3.74
- (E) 5.71

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- 82. The base of a solid is a region enclosed by the circle  $x^2 + y^2 = 4$ . What is the approximate volume of the solid if the cross sections of the solid perpendicular to the x-axis are semicircles?

- 83. The temperature of a cup of coffee is dropping at the rate of  $f(t) = 4 \sin\left(\frac{t}{4}\right)$  degrees for  $0 \le t \le 5$ , where f is measured in Fahrenheit and s in minutes. If initially, the coffee is 95°F, find its temperature to the nearest degree Fahrenheit 5 minutes later.
  - (A) 84
- (C) 91
- (D) 92 (E) 94
- 84. The graphs of f', g', p', and q' are shown in Figure 2T-9. Which of the functions f. g. p. or q have a relative minimum on (a, b)?
  - (A) fonly (B) g only
- - (C) p only (D) q only
  - (E) q and p only

- 85. What is the volume of the solid obtained by revolving the region enclosed by the graphs of  $x = y^2$  and x = 9 about the y-axis?

- 86. At what value(s) of x do the graphs of  $y = e^x$ and  $y = x^2 + 5x$  have parallel tangent lines?
  - (A) -2.5
- (B) 0
- (C) 0 and 5
- (D) -5 and 0.24
- (E) -2.45 and 2.25
- 87. Let y represent the population in a town. If y decreases according to the equation  $\frac{dy}{dt} = ky$ , with s measured in years, and the population decreases by 25% in 6 years, then k=

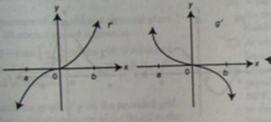
  - (A) -8.318 (B) -1.726 (C) -0.231
  - (D) -0.120 (E) -0.048
- 88. If  $h(x) = \int (x-5)^3 dx$  on [4, 8], then h has a
  - (A) 4 (D) 7
- (C) 6

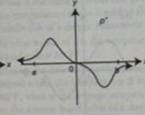
(E) 8

- 84. The graphs of f', g', p', and q' are shown in Figure 2T-9. Which of the functions f. g. p. or q have a relative minimum on (a, b)?

  - (A) fonly (B) gonly

  - (C) p only (D) q only
  - (E) q and p only
- population decreases by 25% in 6 years, then k=
- (A) -8.318 (B) -1.726 (C) -0.231 (D) -0.120 (E) -0.048
- 88. If  $h(x) = \int (t-5)^3 dt$  on [4, 8], then h has a
  - (A) 4 (D) 7
- (B) 5 (E) 8
- (C) 6





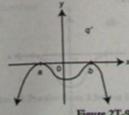


Figure 2T-9

- 89. The volume of the solid generated by revolving the region bounded by the graph of  $y = x^3$ , the line y = 1, and the y-axis about the y-axis is
  - (A)  $\frac{\pi}{4}$
- (B)  $\frac{2\pi}{5}$
- (C)  $\frac{3\pi}{5}$

- (D)  $\frac{2\pi}{3}$
- (E)  $\frac{3\pi}{4}$
- 90. If  $p(x) = \int_a^x q(t)dt$  a < x < b and the graph of q is shown in Figure 2T-10, which of the graphs shown in Figure 2T-11 is a possible graph of p?

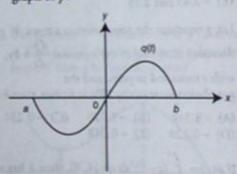


Figure 2T-10

- 91. If f(x) = -|x-3|, which of the following statement about f is true?
  - I. f is differentiable at x=3.
  - II. f has an absolute minimum at x=3.
  - III. f has a point of inflection at x=3.
    - (A) II only
    - (B) III only
    - (C) II and III only
    - (D) I, II, and III
    - (E) None
- 92. The equation of the tangent line to the graph of  $y = \sin x$  for  $0 \le x \le \pi$  at the point where  $\frac{dy}{dx} = \frac{1}{1}$  is

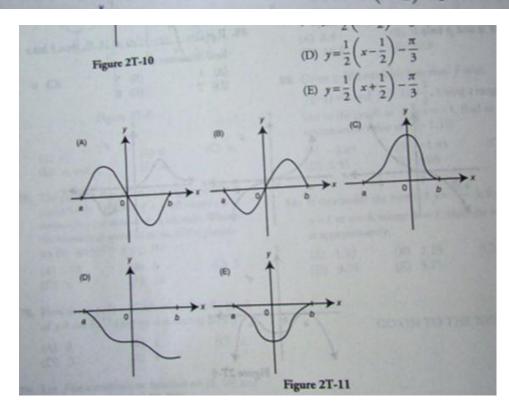
(A) 
$$y = \frac{1}{2} \left( x - \frac{\pi}{3} \right) - \frac{\sqrt{3}}{2}$$

(B) 
$$y = \frac{1}{2} \left( x - \frac{\pi}{3} \right) + \frac{\sqrt{3}}{2}$$

(C) 
$$y = \frac{1}{2} \left( x - \frac{1}{2} \right) + \frac{\pi}{3}$$

(D) 
$$y = \frac{1}{2} \left( x - \frac{1}{2} \right) - \frac{\pi}{3}$$

(E) 
$$y = \frac{1}{2} \left( x + \frac{1}{2} \right) - \frac{\pi}{3}$$



## Section II-Part A

No. of the last of		THE RESERVE OF THE PERSON NAMED IN
Number of Questions	Time	Use of Calculator
2	30 Minutes	Yes

### Directions:

Show all work. You may not receive any credit for correct answers without supporting work. You may use an approved calculator to help solve a problem. However, you must clearly indicate the setup of your solution using mathematical notations and not calculator syntax. Calculators may be used to find the derivative of a function at a point, compute the numerical value of a definite integral, or solve an equation. Unless otherwise indicated, you may assume the following: (a) the numeric or algebraic answers need not be simplified; (b) your answer, if expressed in approximation, should be correct to 3 places after the decimal point; and (c) the domain of a function f is the set of all real numbers.

- Let R be the region in the first quadrant enclosed by the graph of y = 2 cos x, the x-axis, and the y-axis.
  - (A) Find the area of the region R.
  - (B) If the line x=a divides the region R into two regions of equal area, find a.
  - (C) Find the volume of the solid obtained by revolving region R about the x-axis.
  - (D) If R is the base of a solid whose cross sections perpendicular to the x-axis are semicircles, find the volume of the solid.
- The temperature of a liquid at a chemical plant during a 20-minute period is given as
  g(t) = 90 4 tan (t)/(20), where g(t) is measured in degrees Fahrenheit, 0 ≤ t ≤ 20 and t is measured in minutes.
  - (A) Sketch the graph of g on the provided asid
- (D) If K is the base of a solid whose cross sections perpendicular to the x-axis are semicircles, find the volume of the solid.
- The temperature of a liquid at a chemical plant during a 20-minute period is given as
  g(t)=90-4 tan (t/20), where g(t) is measured in degrees Fahrenheit, 0 ≤ t ≤ 20 and t is measured in minutes.
  - (A) Sketch the graph of g on the provided grid. What is the temperature of the liquid to the nearest hundredth of a degree Fahrenheit when t= 10? (See Figure 2T-12.)

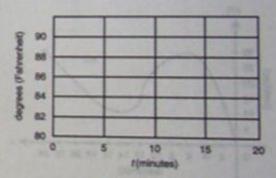


Figure 2T-12

- (B) What is the instantaneous rate of change of the temperature of the liquid to the nearest hundredth of a degree Fahrenheit at r= 10?
- (C) At what values of r is the temperature of the liquid below 86°F?
- (D) During the time within the 20-minute period when the temperature is below 86°F what is the

### Figure 2T-12

- (B) What is the instantaneous rate of change of the temperature of the liquid to the nearest hundredth of a degree Fahrenheit at t= 10?
- (C) At what values of t is the temperature of the liquid below 86°F?
- (D) During the time within the 20-minute period when the temperature is below 86°F, what is the average temperature to the nearest hundredth of a degree Fahrenheit?

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## Section II-Part B

Number of Questions	Time	Use of Calculator
4	60 Minutes	No

#### Directions:

The use of a calculator is not permitted in this part of the exam. When you have finished this part of the exam, you may return to the problems in Part A of Section II and continue to work on them. However, you may not use a calculator. You should thou all work. You may not receive any credit for correct answers without supporting work. Unless otherwise indicated, the numeric or algebraic answers need not be simplified, and the domain of a function f is the set of all real numbers.

A particle is moving on a coordinate line. The graph of its velocity function v(t) for 0 ≤ t ≤ 24 seconds is shown in Figure 2T-13.

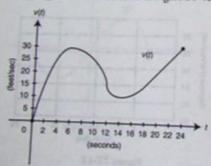


Figure 2T-13

- (C) find  $\lim_{x\to\infty} f(x)$  and  $\lim_{x\to\infty} f(x)$ .
- (D) find the absolute maximum value of f and justify your answer.
- (E) show that if  $f(x) = ae^{-bx^2}$  where a > 0 and b > 0, the absolute maximum value of f is a.
- 5. The function f is defined as  $f(x) = \int_0^x g(t)dt$  where the graph of g consists of five line segments as shown in Figure 2T-14.
  - (A) Find f(-3) and f(3).
  - (B) Find all values of x on (−3, 3) such that f has a relative maximum or minimum. Justify your answer.

### Figure 2T-13

- (A) Using midpoints of the three subintervals of equal length, find the approximate value of  $\int_{0}^{34} v(t) dt$ .
- (B) Using the result in part (A), find the average velocity over the interval 0 ≤ t ≤ 24 seconds.
- (C) Find the average acceleration over the interval 0 ≤ t ≤ 24 seconds.
- (D) When is the acceleration of the particle equal to zero?
- (E) Find the approximate acceleration at t=20 seconds.

Given the function  $f(x) = 3e^{-2x^2}$ ,

- (A) at what value(s) of x, if any, is f'(x) = 0?
- (B) at what value(s) of x, if any, is f''(x) = 0?

Justify your answer.

- (C) Find all values of x on (-3, 3) such that the graph f has a change of concavity. Justify your answer.
- (D) Write an equation of the line tangent to the graph to f at x=1.

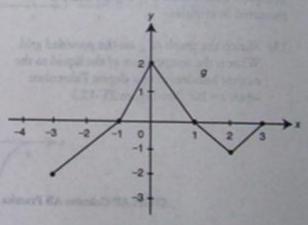


Figure 2T-14

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- 6. The slope of a function f at any point (x, y) is  $\frac{y}{2x^2}$ . The point (2, 1) is on the graph of f.
  - (A) Write an equation of the tangent line to the graph of f at x=2.
- (B) Use the tangent line in part (A) to approximate f(2.5).
- (C) Solve the differential equation  $\frac{dy}{dx} = \frac{y}{2x^2}$  with the initial condition f(2) = 1.

4.0

(D) Use the solution in part (C) and find f(2.5).

STOP. AP Calculus AB Practice Exam 2 Section II-Part B

Answers to AB Practice Exam 2—Section I				
Constitution of the				
Part A	12.C	24. C	81. C	
1. B	13. C	25. A	82. B	
2.A	14.A	26. B	83. A	
3. C	15. C	27. E	84. A	
4. D	16. E	28. D	85. E	
5. E	17. A		86. E	
6. A	18. A	Part B	87. E	
7. B	19. A	76. E	88. B	
8. D	20. E	77. E	89. C	
9. C	21. C	78. D	90. E	
10. A	22. D	79. D	91. E	
11. D	23. C	80. D	92. B	

#### Answers to AB Practice Exam 2—Section II Part A Part B (3 pts.) (2 pts.) 1. (A) 2 3. (A) 480 (2 pts.) (3 pts.) (B) a= -(B) 20 ft/s (1 pc.) (C) 1.25 fds2 (C) x2 (2 per.) (2 pts.) (D) t=6 and t=14 (D) $\frac{\pi^2}{8}$ (1 pt.) (2 pts.) (E) 2.5 ft/s2 (1 pt.) 2. (A) See Figure 2TS-16 in solution 4. (A) x=0 (3 pts.) (2 pts.) and g(10) = 87.82°. (B) x= 主 (2 pts.) (B) -0.26° (2 pts.) (2 pts.) (C) 15.708 < \$ ≤ 20 (2 pts.) (2 pts.) (D) 3 (D) 84.99° (2 pts.) (E) See solution. (2 pts.) 5. (A) f(-3)=1 and f(3)=0(3 pts.) (B) x=-1, 1(2 pts.) (C) x=0 and x=2(2 pts.) (D) y=1 (3 pts.) 6. (A) y= (1 pt.) (C) $y = e^{(-1/2s)+(1/4)}$ (4 pts.) (1 pt.) (D) e1/20 (or 1.051)

Practice the Skill not the solutions.